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John L Hargrave

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1834

A SHORT PRACTICAL TREATISE ON SPHERICAL TRIGONOMETRY;

CONTAINING A FEW SIMPLE RULES,

BY WHICH

THE GREAT DIFFICULTIES TO BE ENCOUNTERED BY THE STUDENT IN THIS BRANCH OF MATHEMATICS ARE EFFECTUALLY OBVIATED.

BY OLIVER BYRNE,

PROFESSOR OF MATHEMATICS;

AUTHOR OF "THE ELEMENTS OF EUCLID, BY COLOURS," "A TREATISE ON ALGEBRA," "LOGARITHMS," &c. &c.

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1835.

TO

V. T. HURTADO, ESQ.,

THIS WORK

IS

MOST RESPECTFULLY DEDICATED

BY

HIS HUMBLE SERVANT,

THE AUTHOR.

PREFACE.

It will perhaps be wondered at, after so much has been written on this subject, that I should have the temerity to offer my mite to the notice of the public, or to imagine myself capable of throwing any light upon a branch of Mathematics that has been so much illuminated by the works of the learned. I trust, however, that when the reader has heard my explanations, he will not consider that I have engaged in a useless or unprofitable undertaking. Much indeed has been written by learned and experienced Mathematicians, but their works seem rather addressed to the proficient, than to the uninitiated student in the science. Their long formulæ, complicated rules, and demonstrations, &c., perplex rather than instruct the beginner, who perhaps, terrified at the mass of difficulties before him, gives up in disgust the study of a subject, which, treated in a simple manner, he had easily acquired. The great fault of all elementary treatises on this branch appears to be the crowding too much upon the mind

of the students, and distracting their attention with useless rules and demonstrations, which retard rather than assist their progress. This I have found by experience, both in my early studies, and in the extensive practice I have since obtained in communicating knowledge to others; and I have frequently in one hour's conversation enabled a pupil to master a subject which he had in vain attempted to acquire by the perusal of the ordinary rules. In short, I felt that a plain practical Treatise on Spherical Trigonometry, in which the pupil's attention should not be distracted from the subject before him, and in which the rules should be as simple and concise as possible, would much facilitate the acquirement of the subject. I have therefore written in the same manner in which I should have explained it by oral communication with my pupils. The formulæ which I have constructed for solving each Case in Spherical Trigonometry will, I hope, obtain the sanction of the Mathematician for their correctness, and the approbation of the student for the ease and rapidity with which they enable him to master the science.

Oliver Byrne.

INTRODUCTION.

By the word Sphere is generally understood any circular body; but the term was appropriated by the ancients to an assemblage of circles and constellations representing their "Primum Mobile." The invention of this Sphere is ascribed to various persons, but it is evidently too remote to be traced by any authentic history. The Chinese had a knowledge of the Sphere at a very early period,* from whom it was probably transmitted to the Chaldeans, thence into Egypt and Greece; but it

* "Xuni, 2400 ans avant J. C. fit faire une sphère d'or enrichie de pierreries où l'on avait les sept planètes et la terre au milieu."—*Historie de La Chine, par Martin*, page 76.

was most successfully studied in the famous school of Alexandria. Here Euclid, the great geometrician, wrote a Treatise on the Sphere, entitled *The Phenomena*, which explained the most interesting parts of ancient astronomy; such as the right oblique ascension of the heavenly bodies, with the various other phenomena, which arise from the apparent diurnal revolution of the Primum Mobile. This work is supposed to be the first on the subject perfectly geometrical: it served long after as a model for other performances on the subject, and is still in existence, but very scarce.

Hipparchus, who flourished about two centuries after Euclid, and one before the Christian era, is said to have laid the foundation of Spherical Trigonometry. In succeeding ages it was improved by Ptolemy, Theodosius, and others; and much is ascribed to Geber, a learned Spaniard, who lived in the sixteenth century.

Baron Napier, however, made the most considerable improvements by his proposition of

circular parts, and his invention of Logarithms,* and up to the present time many works of great merit have appeared in Europe. Yet the present, although the last, the author hopes will be found not the least useful.

* The author has discovered so simple a method of constructing Logarithms, that they may be calculated with as much facility as the most trifling question in common arithmetic, thus superseding the necessity of tables.

SPHERICAL TRIGONOMETRY.

DEFINITIONS.

I.

A sphere is a solid, such that if it be cut by a plane in any position the section will be a circle.

or,

It is formed by the revolution of a semicircle about the diameter which remains fixed.

or,

It is a solid such that all lines drawn from a point within called the centre to the surface are equal to one another.

II.

The circles cutting the sphere are divided

into two kinds, the greater and lesser circles of the sphere: the greater passes through the centre, the less does not.

III.

The nearest distance between any two points on the sphere, is the arc of a great circle;—for such an arc being described with the greatest radius, is less curved than the arc of any small circle.

IV.

A spherical triangle is formed by three great circles on the surface of the sphere.

V.

A spherical angle is formed by the inclination of two great circles on the surface of the sphere meeting in a point, called the angular point.

VI.

Spherical triangles are also distinguished as right-angled, quadrantal and oblique:—thus

when one of the angles is 90° it is called right-angled.

VII.

Any obtuse-angled spherical triangle may be divided into two right-angled spherical triangles, by letting fall from any of the angular points on the opposite side a great circle, whose plane will be perpendicular to the plane of the base.

OF RIGHT-ANGLED SPHERICAL TRIGONOMETRY.

Right-angled triangle spherical trigonometry may be divided into the six following cases:—

I. When the hypothenuse and a leg,
II. the hypothenuse and an angle,
III. a leg and its opposite angle,
IV. a leg and its adjacent angle,
V. two legs,
VI. two angles,

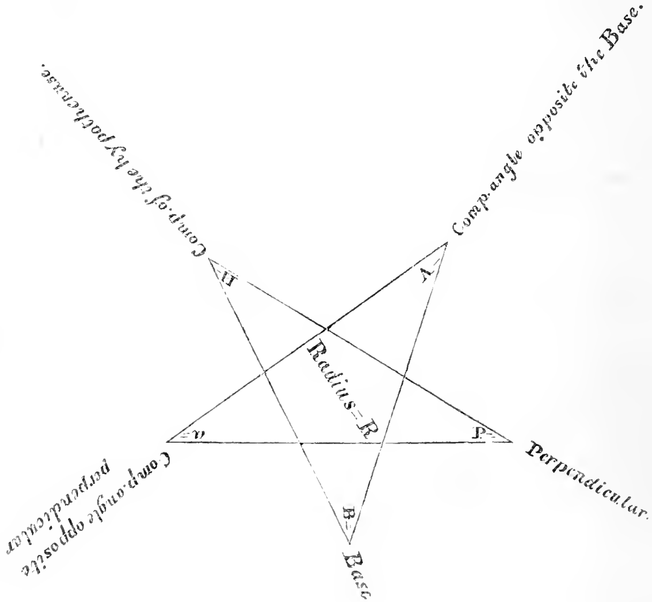
are given together with the right angle.

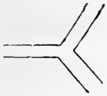
To find the remaining parts of the right-angled spherical triangle,

Place on the angular points of the annexed

five-sided figure (its construction is simple) first the base, next the perpendicular, thirdly the complement of the angle opposite the base, fourthly the complement of the hypothenuse, fifthly the complement of the angle opposite the perpendicular, and in the centre radius as in the annexed diagram.

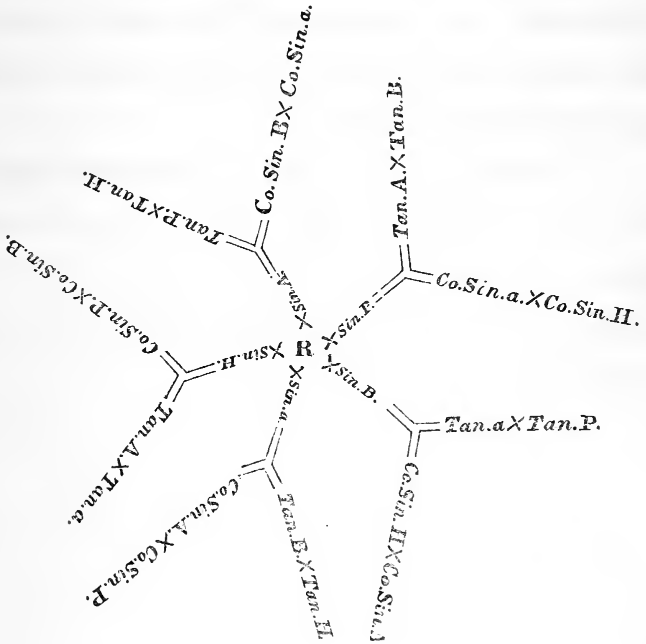
R, Radius: B, Base: P, Perpendicular: A, Complement of the angle opposite the base: H, complement of the hypothenuse: a, Complement of the angle opposite the perpendicular.





This sign signifies that the quantities standing on the three prongs are equal.

Then we have the following general theorem.

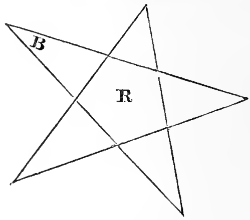


The above formula solves all the cases in right-angled spherical trigonometry, but it is much better to solve each case separate.

RULE FOR THE ARRANGEMENT OF ALL THE CASES.

Place in their respective places the given parts, and then refer to the general formula for the solution of the case.

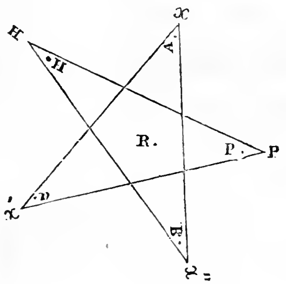
[Obs.] This diagram, so constant in use, does not require Geometrical exactness, nor yet right lines, as it is only to regulate the arrangement of B. P. A. H. *a.* and R; it is no



matter which of the angles we place B. in, so it be followed right or left by P. A. H. and *a*.

CASE I.

WHEN THE HYPOTHENUSE AND A LEG ARE GIVEN TO FIND THE REST.



First suppose the leg given to be the perpendicular, find the part which is represented by x on the diagram, which stands for the complement of the angle opposite the base.

According to the formula we have R.×Sin. x= Tan. H.×Tan. P.

Then Sin. x.$=\dfrac{\text{Tan. H.}\times\text{Tan. P.}}{\text{R}}$

Log. Sin. x=Log. Tan. H.+Log. Tan. P. —Log. R.

Given the hypothenuse 63° 57′ 7″ ∴ H. 26° 2′ 53″.

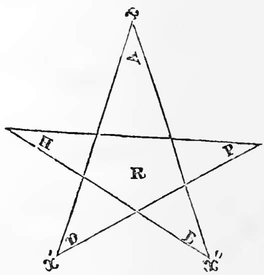
Given the perpendicular 40° 0′ 0″ P.

The solution is as follows:

9.6894258+9.9238135—10.0000000=9.6132393.=Sin. x=24° 13′ 55.″ but x is the complement of the angle opposite the base, then the angle opposite the base is 65° 46′ 5″.

Next find the part which is represented by x', which stands for the complement of the angle opposite the perpendicular.

The solution of this first Case will be the same, if the side given was the base.



R.×Sin. P.=Co. Sin. H.×Co. Sin. x'.

Consequently we have Co. Sin. $x' = \dfrac{\text{R.}\times\text{Sin. P.}}{\text{Co. Sin. H.}}$

or which is the same,

Log. Co. Sin. x'=Log. R.+Sin. P.—Log. Co. Sin. H.

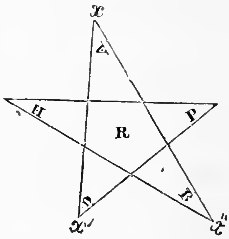
EXAMPLE.

The solution is obtained by substituting in this formula the logarithms of R. Sin. P. and Co. Sin. H.; then: Log. Co. Sin. $x'=10.+9.8080675-9.9534206=9.8546469=45^{\circ}\ 41'\ 21''$: but the Co. Sin. of x' is the sine of the angle opposite the perpendicular .·. $45^{\circ}\ 41'\ 21''$ the angle required.

To find the part represented by x'' which stands for the base.

R.×Sin. H.=Co. Sin. P.×Co. Sin. x''

$$\text{Co. Sin. } x''=\frac{\text{R.}\times\text{Sin. H.}}{\text{Co. Sin. P.}}$$



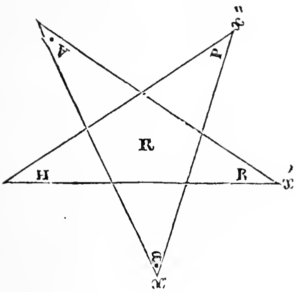
Then we have, Log. Co. Sin. x''=Log. R.+Log. Sin. H. —Log. Co. Sin. P.

NUMERAL SOLUTION.

Log. Co. Sin. $x''=10.+9.642846-9.884254=9.7585924=55^{\circ}$. The leg represented by x''.

CASE II.

WHEN THE HYPOTHENUSE AND AN ANGLE ARE GIVEN TO FIND THE REMAINING PARTS OF THE RIGHT-ANGLED SPHERICAL TRIANGLE.



We may either take the angle opposite the base or opposite the perpendicular, and the solution will be the same.

Let it first be required to find the value of x which stands for the complement of the angle, opposite the perpendicular.

Then according to the general formula we have R.×Sin. H.=Tan. A.×Tan. x.

Then R.$\times$Sin. H.$=$Tan. A.$\times$Tan. x.

$$\text{Tan. } x.=\frac{\text{R.}\times\text{Sin. H.}}{\text{Tan. A.}}$$

Then is Log. of Tan. x.$=$Log. R.$+$Log. Sin. H.—Tan. A. Let the hypothenuse be$=63^\circ\ 56'\ 7''$ and the given angle$=45^\circ\ 41'\ 21''$. H.$=26^\circ\ 3'\ 53''$ or $90^\circ—63^\circ\ 56'\ 7''$; A.$=44^\circ\ 18'\ 39''=90^\circ—45^\circ\ 41'\ 21''$.

Log. Tan. x.$=10.+9.6428464—9.9895514.$

$$= \quad 9.6532950$$

x. $=34^\circ\ 13'\ 55''$, the complement of angle opposite the perpendicular; the angle$=65\ 46'\ 5''$.

Next let the required part be x', which is situated in place of the base.

In each of the cases a reference to the general formula is given; and another thing may be remembered here, that the quantity required, and the two that are given, must be so arranged that two of the known quantities must be multiplied and equalled with the other known quantity; R. the radius is always known.

R.$\times$Sin. $x'$$=$Co. Sin. H.$\times$Co. Sin. A.

$$\text{Sin. } x' = \frac{\text{Co. Sin. H.} \times \text{Co. Sin. A.}}{\text{R.}}$$ Therefore

Log. Sin. x'. = Log. Co. Sin. H. + Log. Co. Sin. A.—Log. R.

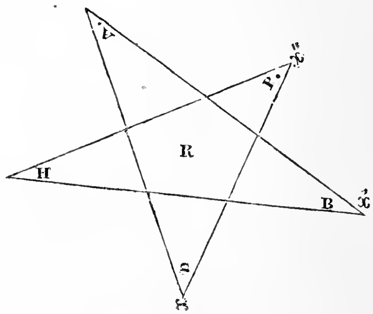
EXAMPLES.

1. Given the hypothenuse of a right-angled spherical triangle 45° 17′, and the angle opposite the base 16° 27′, to find the base?

2. Required the base of a right-angled spherical triangle, when the hypothenuse is = 103° 30′, and the angle opposite the base = 75° 35′?

Next, let it be required to find the value of x'', which stands in the diagram in the place of the perpendicular.

R. × Sin. A. = Tan. H. × Tan. x''. Consequently, we have the value of x'', or its tangent $= \dfrac{\text{R.} \times \text{Sin. A.}}{\text{Tan. H.}}$ by Logarithms, thus Log. of x'' = Log. R. + Sin. of A — Log. Tan. H.



1. Given the hypothenuse $63^\circ\ 53'\ 7''$, and the angle opposite the perpendicular $45^\circ\ 41'\ 21''$, to find the perpendicular?

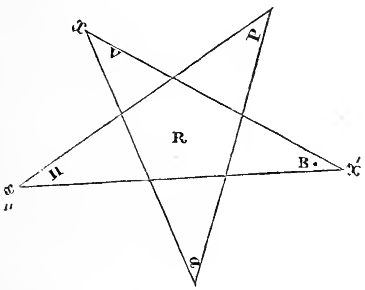
Answer 40°.

Required the perpendicular of a right-angled spherical triangle, when the hypothenuse is 57° and the angle 30°?

CASE III.

WHEN A LEG AND ITS OPPOSITE ANGLE ARE GIVEN TO FIND THE REMAINING PARTS.

Let the perpendicular and the angle opposite it be given; which is the same as the base, and the angle opposite the base; for the perpendicular may be considered the base when we please. 1st. to find the value of x, which represents the complement of the angle opposite the base.

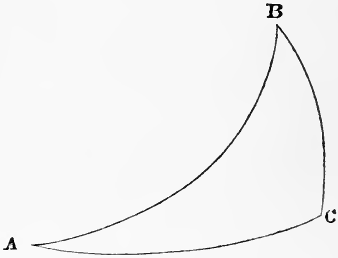


$$R.\times \text{Sin. } a. = \text{Co. Sin. } x \times \text{Co. Sin. P.}$$

$$\text{Co. Sin. } x = \frac{R.\times \text{Sin. } a.}{\text{Co. Sin. P.}} \text{ or Log. Co. Sin. } x =$$

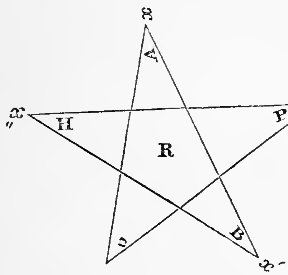
Log. R.+Log. Sin. a.—Log. Co. Sin. P. Suppose the perpendicular $=55^{\circ}\ 0'$, and its opposite angle $=65^{\circ}\ 46'\ 5''$, then we have P. $=55^{\circ}\ 0'\ 0''$ and $a.=$ Comp. of $65^{\circ}\ 46'\ 5''=24^{\circ}\ 13'\ 55''$. Consequently Log. Co. Sin. $x.=10.+9.6132407$ $-9.7585913=9.8546494$, the Log. Co. Sin. of x; $\therefore$ $x=45^{\circ}\ 41'\ 21''$.

Suppose the perpendicular BC to be $59^{\circ}\ 30'$, and its opposite angle BAC $67^{\circ}\ 47'$, to find the angle opposite the base?



To find x' which stands for the other leg in the general scheme. (Page 4.)

From the general formula we have R.$\times$Sin. x' $=$Tan. $a.\times$Tan. P. Sin.



$x'=\dfrac{\text{Tan. } a.+\text{Tan. P.}}{\text{R.}}$ as before P. $=55^{\circ}\ 0'\ 0''$ and $a.=24^{\circ}\ 13'\ 55''$. Log. Sin. $x'=$ Log. Tan. $a.+$ Log. Tan. P.—Log. R.

Log. Sin. $x'=10.1547732+9.6532976-10$.

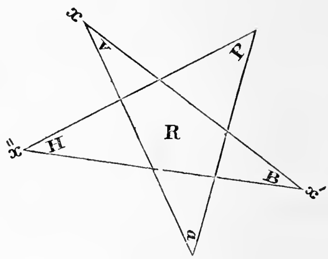
$\therefore$ The Log. Co. Sin. $x' = 9.8080708 = 40^{\circ}\ 0'\ 0''$.

Required to find the base AC of the above right-angled spherical triangle, when the perpendicular BC is $23^{\circ}\ 57'$, and its opposite angle $38^{\circ}\ 42'\ 3''$.

In order to find the value of x which stands in the scheme for the complement of the hypothenuse.

R. $\times$ Sin. P. $=$ Co. Sin. a. $\times$ Co. Sin. x''. Co.

$$\text{Sin. } x'' = \frac{\text{R.} \times \text{Sin. P.}}{\text{Co. Sin. A.}}$$



or Log. Co. Sin. $x'' =$ Log. R. $+$ Log. Sin. P. $-$ Log. Co. Sin. a. Then as before P. $= 55^{\circ}\ 0'\ 0''$ a $24^{\circ}\ 13'\ 55''$

Log. Co. Sin. $x'' = 10. + 9.9133645 - 9.9599432$. Log. Co. Sin. $x'' = 9.9534213$. Therefore $x'' = 26^{\circ}\ 3'\ 53''$. And the hypothenuse $= 63^{\circ}\ 56'\ 7''$.

Given the perpendicular BC of a right-angled spherical triangle $= 55^{\circ}$, and the angle opposite the perpendicular CAB $= 28^{\circ}\ 15'$, to find the hypothenuse?

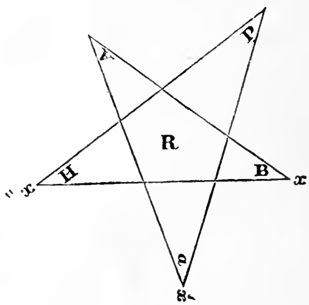
Answer.

CASE IV.

A LEG AND ITS ADJACENT ANGLE BEING GIVEN TO FIND THE REST.

Suppose the given leg $= 55^\circ\ 0'\ 0''$, and its adjacent angle $= 45^\circ\ 41'\ 21''$, which in the diagram stands for the perpendicular, and its adjacent angle.

P. $= 55^\circ\ 0'\ 0''$; A. $= 44^\circ\ 18'\ 39''$, the complement of the angle opposite the base.



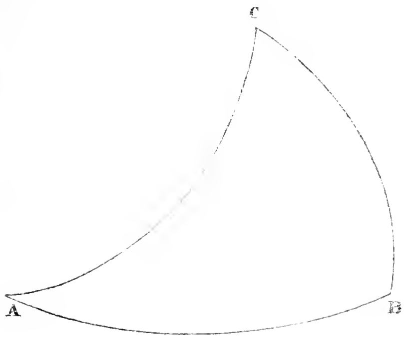
1st, find x which stands for the base. R. $\times$ Sin. P. $=$ Tan. A. $\times$ Tan. x

$$\text{Tan. } x. = \frac{\text{R.} \times \text{Sin. P.}}{\text{Tan. A.}}$$

Log. Tan. $x.$ = Log. R. + Log. Sin. P. — Log. Tan. A. Log. Tan. $x.$ $= 10 + 9.9133645 - 9.$

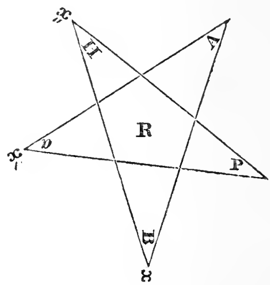
9895514 = 9.9238131 = Log. Tan. of x; $\therefore$ x = 40°.

2. Suppose the given leg BC. = 57° 30′, and its adjacent angle BCA. = 45° 38′, to find the base?



Next find the part represented by x', which stands for the complement of the angle opposite the perpendicular.

$$R. \times Sin.\ x' = Co.\ Sin.\ P. \times Co.\ Sin.\ A.\ \therefore\ x' = \frac{Co.\ Sin.\ P. \times Co.\ Sin.\ A.}{R.}$$



Or by logarithms, Log. Sin. x' = Log. Co. Sin. P. + Log. Co. Sin. A. — Log. R. Log. Sin. x' = 9.7585913 + 9.8546465 — 10. Log. Sin. x' = 9.6132378 $\therefore$ x' = 24° 13′ 55″, the complement of which is the required angle = 65° 46′ 5″.

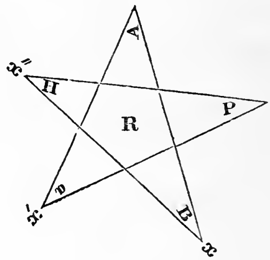
2nd. If the perpendicular be 60°, and the

angle opposite the base 30°, require the angle opposite the perpendicular?

3rd. Require the angle opposite the perpendicular, when the angle opposite the base is $=45^\circ$, and the perpendicular 35°?

Then find x'', which is the complement of the hypothenuse.

R. $\times$ Sin. A. $=$ Tan. $x'' \times$ Tan. P. Tan. $x'' = \dfrac{\text{R.} \times \text{Sin. A.}}{\text{Tan. P.}}$ or Log. Tan. $x''=$ Log. R. $+$ Log. Sin. A. — Log. Tan. P. Log. Tan. $x'' = 10. + 9.8441979 - 10.1547732$. Log. Tan. $x'' = 9.6854247$ $\therefore$ $x'' = 26^\circ\ 2'\ 53''$.



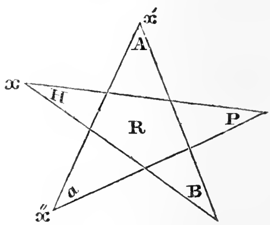
$$\begin{array}{rrrr} \text{Then from} & 90^\circ & 0' & 0'' \\ \text{Take} & 26^\circ & 2' & 53'' \\ \hline & 63^\circ & 57' & 7'' \end{array} = \text{the hypothenuse.}$$

2. Given the perpendicular $=98^\circ$, and the angle opposite the base $=32^\circ$, to find the hypothenuse.

CASE V.

WHEN THE TWO LEGS ARE GIVEN TO FIND THE HYPOTHENUSE AND ANGLES.

To find x, which represents the complement of the hypothenuse, from whence the hypothenuse is easily known.



R.$\times$Sin. x = Co. Sin. P.$\times$ Co. Sin. B. whence the

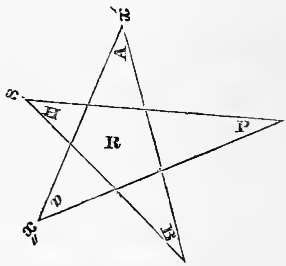
Sin. $x. = \dfrac{\text{Co. Sin. P.}\times\text{Co. Sin. B.}}{\text{R.}}$; or Log. Sin. x = Log. Co. Sin. P.+Log. Co. Sin. B.—Log. R. That is Log. Sin. $x = 9.7585913 + 9.8842540 - 10 = 9.6428453$ = Log. Co. Sin. x. $\therefore = 26^\circ\ 3'\ 53''$. Consequently the hypothenuse $= 63^\circ\ 56'\ 7''$.

2. Given the base $=61^\circ$, and the perpendicular $=80^\circ$, require the hypothenuse?

3. Required the hypothenuse when the base is 90°, and the perpendicular $23^\circ\ 28'$?

4. When the perpendicular is $37^\circ\ 42'$, and the base $49^\circ\ 50'$, what is the hypothenuse?

Then find x', which stands in the diagram for the complement of the angle opposite the base.



R.$\times$Sin. P. $-$ Tan. B. $\times$Tan. x'. Consequently

$$\text{Tan. } x' = \frac{\text{R.}\times\text{Sin. P.}}{\text{Tan. B.}}; \text{ or}$$

Log. Tan. $x'=$Log. R.$+$Log. Sin. P.$-$Log. Tan. B.

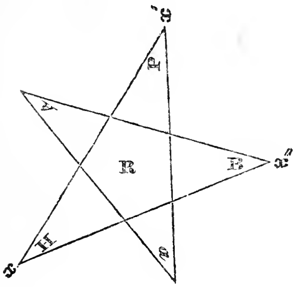
Log. Tan. $x'=10.+9.9133645-9.9238135=9.9895510=$ The Log. Tan. of x' $\therefore$ $x'=44^\circ\ 18'\ 39''$ and $90^\circ-44^\circ\ 18'\ 39''=45^\circ\ 41'\ 21''$, the angle required.

2. Given the base, and perpendicular, $=57^\circ\ 15'$ and $39^\circ\ 30'$ respectively, required the angle opposite the base?

CASE VI.

GIVEN THE TWO ANGLES, TO FIND THE OTHER PARTS OF THE RIGHT-ANGLED SPHERICAL TRIANGLE, OBSERVING THAT THE RIGHT ANGLE IS ALWAYS KNOWN.

Let the angle opposite the base be $= 45^\circ\ 41'\ 21''$ then A. is $= 44^\circ\ 18'\ 39''$ its complement; also let the angle opposite the perpendicular $= 65^\circ\ 46'\ 5''$ its complement $= 24^\circ\ 13'\ 55'' = a$. The values of x, x', x'', may be found in the same manner as in the preceding cases.



The multiplicity of Examples being only given to exercise the student.

OF OBLIQUE-ANGLED SPHERICAL

TRIANGLE TRIGONOMETRY.

Every oblique-angled spherical triangle being composed of six parts,—namely, three sides, and three angles; any three being given, the remaining three may be found.

Let there be given

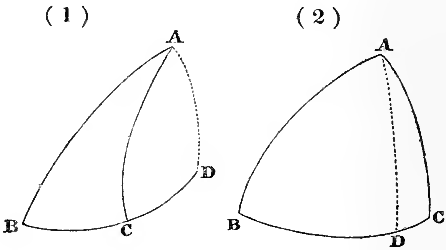
I. Two sides and an opposite angle.
II. Two sides and an included angle.
III. Two angles and an opposite side.
IV. Two angles and an included side.
V. Three sides.
VI. Three angles.

The four first cases may be solved by the formula, page 6, and the other two must have separate rules.

CASE I.

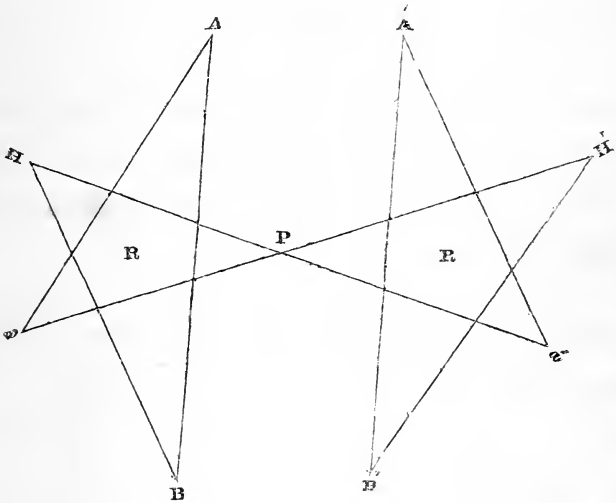
GIVEN TWO SIDES AND AN ANGLE, OPPOSITE ONE OF THEM TO FIND THE REMAINING PARTS OF THE SPHERICAL TRIANGLE.

Given the sides AB, AC, and either of the angles ABC, or ACB, to find the side BC and the remaining angles?



We must suppose a perpendicular AD to be let fall in such a manner, that two of the known parts of the spherical triangle may be in the right-angled spherical triangle BAD or DAC.

Then it is easily observed, that a figure HAH′Ba, which is double of the one given in right-angled spherical triangle Trigonometry, will answer for oblique-angled spherical triangle Trigonometry.



R. = The Radius.

P. = The perpendicular AD.

B. = The base BD.

a. = The complement of the angle B.

H. = The complement of the hypothenuse AB.

A. = The complement of the angle BAB.

B$'$. = The base CD.

a'. = The complement of the angle ACD.

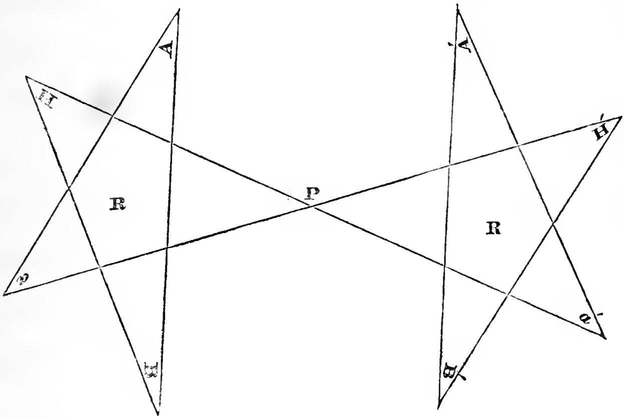
H$'$. = The complement of the hypothenuse AC.

A$'$. = The complement of the angle CAD.

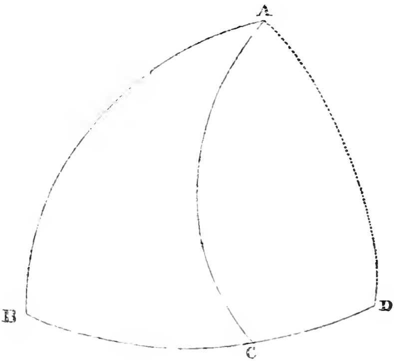
In this case the solution is evident, only to keep in mind that when the perpendicular falls outside the base, as in fig., the (1) $A \backsim A'$ is the angle BAC, and $B \backsim B'$ is the base BC, and the Co. a' is the supplement of the angle ACB, and not the angle itself; but when the perpendicular falls within the spherical triangle as in fig. (2) Co. A+Co. A$'$ is the angle BAC, B+B is the base BC, and Co. a' is the angle ACB.

CASE II.

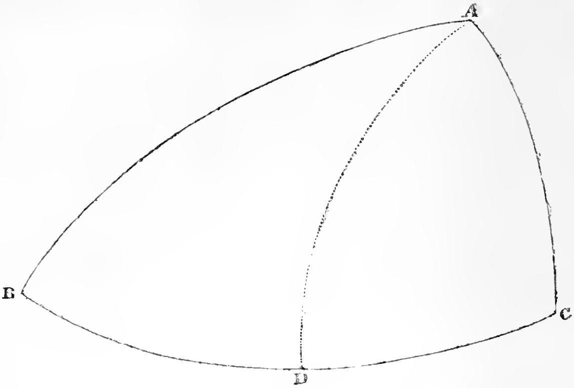
GIVEN ANY TWO SIDES AND THEIR INCLUDED ANGLE OF AN OBLIQUE SPHERICAL TRIANGLE, TO FIND THE REMAINING PARTS OF THE TRIANGLE.



(1)



(2)



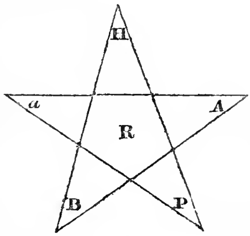
Given H, (or AB,) $B \backsim B'$ or $B + B'$ or (BC); also the angle ABC, to find the side AC, and the angles BAC and BCA.

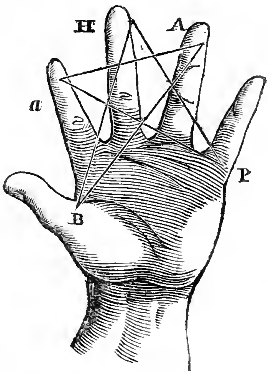
It must be observed that the perpendicular arc AD must be let fall from the point A, on the base BC, or BC, produced; it may be also let fall from the point C, on the great circle AB, or AB, produced.

From what has been said, the solution of the third and fourth cases will easily be arranged and solved; and it may not be amiss to state here again, that $A \backsim A'$ is equal to the angle BAC, when the perpendicular arc AD falls outside the base BC, and $BC = B \backsim B'$; but when the arc AD falls inside of the triangle

Co. A+Co. A′, is = the angle BAC and B+B′ = BC the base.

It happens that the angle ACD is the supplement of the angle ACB, when the perpendicular falls outside the triangle, or Co. *a′* must be taken from 180 degrees, to give the angle ACB.





In order that this system may be more simplified, the diagram given in the work may be imagined as placed upon the fingers of the left hand, as in the annexed scheme:

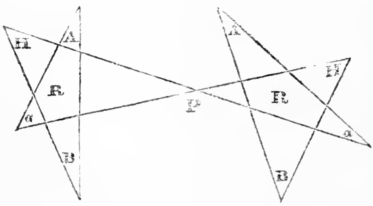
The thumb is marked B, and represents the base.

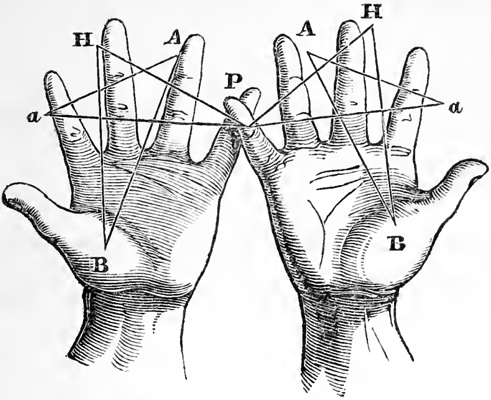
The little finger is marked P, and represents the perpendicular.

The ring-finger is marked A, and represents the complement of the angle opposite the base.

The middle finger is marked H, and represents the complement of the hypothenuse.

And the finger next the thumb represents the complement of the angle opposite the perpendicular, and is marked *a*. Thus the diagram, so familiar in the work, may be conceived to exist on the fingers of our hand, and in short may be slightly marked, until their positions are known.





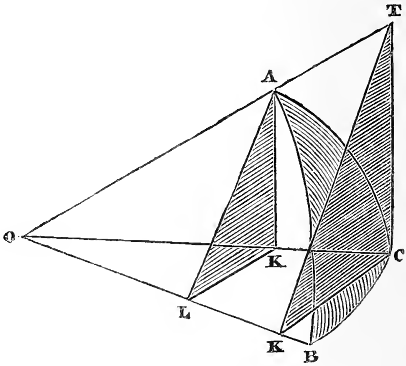
From what has been said of right-angle tri-angled spherical trigonometry, with respect to the left hand, we can easily conceive the mode of arranging both hands (as in the annexed plate) for oblique-angled spherical triangle trigonometry.

DEMONSTRATION

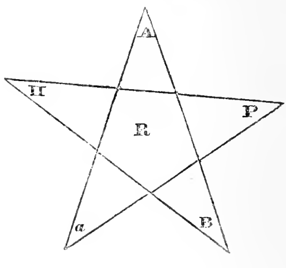
OF THE

FOREGOING FORMULÆ.

The formulæ given page 6 are demonstrated as follow :—



ABC is any right-angled spherical triangle BC, the base AC, the perpendicular CAB, the angle opposite the base; AB, the hypothenuse, and ABC, the angle opposite the perpendicular.



It is a well-known property that Sin. BC. : Co. Tan. B. :: Tan. AC. : Radius (R.) $\therefore$ R.$\times$Sin. BC. = Co. Tan. B.$\times$Tan. AC.

And it is also demonstrable that Sin. AB. : R. :: Sin. BC. : Sin. A. $\therefore$ R.$\times$Sin. BC. = Sin. BA.$\times$Sin. A.

Secondly, The analogy, Co. Sin. B. : Co. Tan. BA. :: Tan. BC. : R. is well known. $\therefore$ R.$\times$Co. Sin. B. = Co. Tan. AB.$\times$Tan. BC.

And Co. Sin. AC. : R. :: Co. Sin. B. : Sin. A. $\therefore$ R.$\times$Co. Sin. B. = Co. Sin. AC.$\times$Sin. A.

Lastly, By two other well-known analogies, Co. Sin. BA. : Co. Tan. B. :: Co. Tan. A. : R. and Co. Sin. AC. : R. :: Co. Sin. BA. : Co. Sin. BC.

$\therefore$ R.$\times$Co. Sin. AB. = Co. Tan. B.$\times$Co. Tan. A. and R.$\times$Co. Sin. AB. = Co. Sin. AC.$\times$ Co. Sin. BC.

Consequently, the general formula page 6 is right.

CASE V.

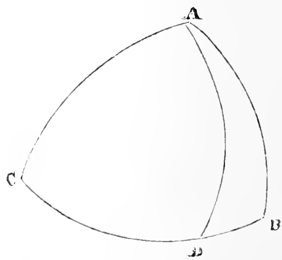
GIVEN THE THREE SIDES TO FIND THE THREE ANGLES.

RULE.

From half the sum of the three sides subtract each of the two sides which contain the required angle.

Then add together the Sines of those two remainders, and the Co-arcs of the Sines of the sides which contain the angle. Half the sum of those four Logs. will give the Sine of half the required angle.

EXAMPLE.



$$\left.\begin{array}{lrrr} AB. = & 79^\circ & 17' & 14'' \\ BC. = & 100^\circ & 0' & 0'' \\ AC. = & 58^\circ & 0' & 0'' \\ \hline 2) & 247^\circ & 17' & 14'' \end{array}\right\}\text{To find the angles.}$$

From $123^\circ\ 38'\ 37'' =$ the $\frac{1}{2}$ Sum

Take $79^\circ\ 17'\ 14''$

1st rem^r. $44^\circ\ 21'\ 23''$

The Sine of which is $= 9.8445513$

Again, from $123^\circ\ 38'\ 37''$

Take $58^\circ\ 0'\ 0''$

2nd rem^r. $65^\circ\ 38'\ 37''$

The Sine of which is $= 9.9595173$

The Co. Arc. Sin. of $58^\circ\ 0'\ 0'' = 0.0715795$

The Co. Arc. Sin. of $79^\circ\ 17'\ 14'' = 0.0076359$

2) 19.8832840

9.9416420

Sine $60^\circ\ 57'\ 28''$

2

$121^\circ\ 54'\ 56''$ = the angle A.

CASE VI.

WHEN THE THREE ANGLES ARE GIVEN TO FIND THE THREE SIDES.

RULE.

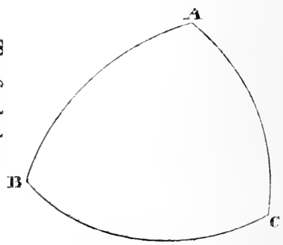
From half the sum of the three angles subtract each of the angles next the required side.

Then add together the Co. Sines of those two remainders, and the Co-Arc of the Sines of each of the adjoining angles. Half the sum of those four Logs. will give the Co. Sin. of half the required side.

EXAMPLE.

$$\left.\begin{array}{rr} \text{Given A.} = & 121^\circ\ 54'\ 56'' \\ \text{B.} = & 50^\circ\ \ 0'\ \ 0'' \\ \text{C.} = & \underline{62^\circ\ 34'\ \ 6''} \end{array}\right\}\text{To find the sides.}$$

$$2)\ 234^\circ\ 29'\ 2''$$



Then from 117° 14′ 31″ = the ½ Sum

Take 62° 34′ 6″ = the angle C.

1st remainr. 54° 40′ 25″ Co. Sin. = 9.7621032

Again, from 117° 14′ 31″

Take 50° 0′ 0″

2nd remainr. 67° 14′ 31″ Co. Sin. = 9.5875321

The Co. Arc Sin. 50° 0′ 0″ = 0.1157460

The Co. Arc Sin. 62° 34′ 6″ = 0.0518018

Add

2) 19.5171831

9.7585915

Co. Sin. 55° 0′

2

110° 0′ the side BC required.

THE END.